Numerical Study of Mach Number and Thermal Effects on Sound Radiation by a Mixing Layer

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Y. Gervais ‡
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Abstract
Mach number and thermal effects on the mechanisms of sound generation and propagation are investigated in spatially evolving two-dimensional isothermal and non-isothermal mixing layers at Mach number ranging from 0.2 to 0.4 and Reynolds number of 400. A characteristic-based formulation is used to solve by direct numerical simulation the compressible Navier-Stokes equations using high-order schemes. The radiated sound is directly computed in a domain that includes both the near-field aerodynamic source region and the far-field sound propagation. In the isothermal mixing layer, Mach number effects may be identified in the acoustic field through an increase of the directivity associated with the non-compactness of the acoustic sources. Baroclinic instability effects may be recognized in the non-isothermal mixing layer, as the presence of counter-rotating vorticity layers, the resulting acoustic sources being found less efficient in the sound generation. An analysis based on the acoustic analogy shows that the directivity increase with the Mach number can be associated with the emergence of density fluctuations of weak amplitude but very efficient in terms of noise generation at shallow angle. This influence, combined with convection and refraction effects, is found to shape the acoustic wavefront pattern depending on the Mach number.

1 Introduction

In many engineering applications, aerodynamic noise needs to be reduced in order to match new environmental expectations, especially in the context of

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aeronautics. To elaborate efficient strategies of flow control leading to noise reduction, the subtle mechanisms of sound generation and propagation need to be better understood while distinguishing the various influences of physical conditions met in practical situations. Among the fundamental parameters acting on the noise emission from aircraft jet engines, the Mach number and thermal effects can be considered as two major factors. However, how these two types of influences can be combined together and modify acoustic emission and propagation is not fully understood yet, this point deserving more investigation.

The goal of the present work is to investigate numerically the influence of Mach number and temperature on sound radiation from a spatially evolving mixing layer flow. A parametric study is conducted where the acoustic source and the associated sound propagation are directly computed using Direct Numerical Simulation (DNS). Detailed information on the source flow field is provided by DNS which can be helpful to better understand the sound generation process. The direct assessment of the sound is essential, since noise modelling prediction theories inevitably rely on assumptions that are not met in real flows [14]. In particular, the key role of the formation of primary structures (Kelvin-Helmholtz vortices) and their interactions (vortex pairing) on the mechanisms of noise emission has been clearly exhibited in mixing layers by direct sound computation [12, 5]. However, the influence of compressibility and temperature on the acoustic efficiency of these physical processes has been less investigated.

Since a limited range of low Mach numbers is considered in this paper, the compressibility effects on the flow dynamics remain weak (nearly incompressible) while strongly influencing the level of noise production. In the same way, the temperature differences investigated here lead only to moderate effects on the vortex dynamics while affecting more significantly the acoustic generation and propagation. The resulting combinations of Mach number and temperature ratios allow the covering of representative flow regimes observed in typical aircraft jet engines.

The need of high-order accurate numerical schemes for the direct computation of the sound in the context of DNS is widely recognized. The very small inherent dissipative and dispersive nature of these schemes provides an improved representation of a wide range of scales, ensuring better resolution of the shorter energy scales and providing an accurate representation of the acoustic wave propagation. Physically, these requirements are related to the preservation of relevant acoustic wave propagation associated with a fine description of the length scales of sound generation. These expectations are particularly demanding for low Mach number flows where acoustic fluctuations are several orders of magnitude smaller than aerodynamic perturbations. In order to satisfy these accuracy requirements, following the practice of previous authors [12, 2, 17], compact centred sixth-order finite difference schemes [22] are used in this work. However, alternatively to these previous authors,
present DNS are based on the solving of a characteristic-based formulation of the Navier-Stokes equations proposed by [31] for the treatment of compressible flows free from strong shock inside the computational domain (non-conservative approach). For the low Mach number cases considered here, the ability of shock capturing would be superfluous, since the most delicate problem is linked to the treatment of boundary conditions that need to be silent (i.e. without the introduction of spurious numerical noise) when relevant acoustic predictions are expected. The numerical strategy proposed here is to take advantage of the simplicity offered by the characteristic-based formulation for the boundary condition implementation that can be done consistently with the interior numerical treatment. Note however that additional absorption/dissipation techniques (see [11] for a review) are found to be necessary to avoid spurious acoustic waves due to the treatment of numerical boundary conditions.

The organisation of this paper is as follows. Section 2 presents the flow configuration and numerical methods. Isothermal and non-isothermal results are presented in section 3 and 4 respectively, while the Mach number scaling of the acoustic intensity is discussed in the specific section 5. An analysis based on acoustic tools is presented in section 6 to understand the directivity change of sound emission depending on the Mach number. Finally, the main conclusions of the study can be found in section 7.

2 Flow configuration and numerical methods

Here, we consider a spatially developing mixing layer between two parallel streams of velocity and temperature \( (U_1, T_1) \) and \( (U_2, T_2) \). Following this convention, in the rest of this paper, the flow variables in the upper \( (y > 0) \) and lower \( (y < 0) \) stream of the mixing layer are subscripted by 1 and 2 respectively. The schematic view of the flow configuration is displayed in figure 1.

To impose the flow at the inlet of the computational domain, a hyperbolic-tangent velocity profile is used with

\[
u_o(y) = \frac{U_1 + U_2}{2} + \frac{\Delta U}{2} \tanh \left( \frac{2y}{\delta_\omega} \right),
\]

where \( \delta_\omega = \Delta U / |\partial u / \partial y|_{\text{max}} \) is the inflow vorticity thickness and \( \Delta U = U_1 - U_2 \) the velocity difference. The inflow temperature profile is defined using the Crocco-Busemann relation

\[
T_o(y) = \frac{1}{2C_p} \left[ -u_o(y)^2 - U_1 U_2 + u_o(y)(U_1 + U_2) \right] + \frac{1}{\Delta U} \left[ u_o(y)(T_1 - T_2) + (T_2 U_1 - T_1 U_2) \right]
\]
Figure 1: Schematic view of the mixing layer configuration. Near- and far-field regions represented by the vorticity and dilatation fields respectively. Inflow velocity/temperature profiles with mean free-stream values \([U_1, T_1]\) and \([U_2, T_2]\). Computational domain includes a buffer zone with near-field aerodynamic dissipation and far-field acoustic absorption.

In this work, we consider only the velocity ratio \(U_1/U_2 = 2\). The isothermal case refers to a temperature ratio \(T_1/T_2 = 1\) while to investigate a non-isothermal situation, we consider also the case \(T_1/T_2 = 2\). The reference pressure is the same in the two streams with \(p_1 = p_2\).

Only weakly compressible cases are investigated here with a Mach number \(M = \Delta U/c_2\) ranging from 0.2 to 0.4 where \(c_2\) is the sound speed in the low speed \((U_2 < U_1)\) and possibly “cold” region \((T_2 \leq T_1)\). The Reynolds number \(Re_\omega = \rho_2 \Delta U \delta_\omega/\mu_2\) is 400 with a constant dynamic viscosity \(\mu_2\). Note that throughout the paper, numerical values are given using \(\rho_2\) (the density in the low speed stream), \(c_2\) and \(\delta_\omega\) for non-dimensionalisation. The Prandtl number \(Pr = \mu_2 C_p/\lambda\) is 0.75 with a constant conductivity \(\lambda\). For clarity, in table 1 are reported the mean free-stream velocities \(U_1\) and \(U_2\) for each Mach number considered.

The size of the full computational domain and the spatial resolution are reported in table 2. Note that in the non-isothermal case \((T_1/T_2 = 2)\), the computational domain is extended in \(y\). This extension is required to avoid acoustic wave reflections in the hot region where acoustic waves propagate faster with an increased wavelength.

The size of the aerodynamic dissipation and acoustic absorption regions are \(\{440 \leq x \leq 800\} \times \{-120 \leq y \leq 120\}\) and \(\{720 \leq x \leq 800\} \times \{-Ly/2 \leq y \leq Ly/2\}\) respectively.
Mach number \( M = \frac{\Delta U}{c_2} \)

<table>
<thead>
<tr>
<th>Velocity</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_1 )</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>( U_2 )</td>
<td>0.2</td>
<td>0.25</td>
<td>0.3</td>
<td>0.35</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 1: Mean free-stream velocities \( U_1 \) and \( U_2 \) at Mach numbers range: \( 0.2 \leq M \leq 0.4 \).

<table>
<thead>
<tr>
<th>MIXING LAYER</th>
<th>([L_x \times L_y])</th>
<th>([n_x \times n_y])</th>
</tr>
</thead>
<tbody>
<tr>
<td>isothermal</td>
<td>([800 \times 800])</td>
<td>([2071 \times 745])</td>
</tr>
<tr>
<td>non-isothermal</td>
<td>([800 \times 1200])</td>
<td>([2071 \times 947])</td>
</tr>
</tbody>
</table>

Table 2: Dimensions of the full computational domain of the isothermal and non-isothermal mixing layers. Representation of size \([L_x \times L_y]\) and number of grid points \([n_x \times n_y]\) in Cartesian coordinates.

\( L_y/2 \) respectively. In the normal direction, the grid is gradually stretched with \( \Delta y_{\min} = 0.175 \) in the shear layer and \( \Delta y_{\max} = 5.948 \) at the lateral boundaries. In the streamwise direction, the grid is nearly uniform for \( 0 \leq x \leq 440 \) with \( \Delta x_{\min} = 0.255 \) while being slightly stretched further downstream (for \( 440 < x \leq L_x \)) with \( \Delta x_{\max} = 1.600 \) at the outlet.

As governing equations, the characteristic-based formulation of [31] is used. The wave modal structure of these equations is well suited to define straightforwardly non-reflecting boundary conditions. Another advantage of this specific formulation is that it makes easier the use of upwind schemes, as discussed by [31] and tested for direct sound computation by [8]. Here, this potentiality is not exploited through the exclusive use of centred schemes to eliminate any bias due to numerical dissipation. Note that the use of centred schemes has required the use of very refined computational grid to avoid spurious oscillations in the acoustic field [8]. In practice, compact sixth-order finite difference schemes are used for the spatial differentiation [22] while the time marching is ensured by a fourth-order Runge-Kutta scheme.

In order to make the mixing layer break-up faster, get better control of the roll-up and pairing processes while introducing a deterministic character of these primary and secondary instabilities, the shear layer is harmonically forced at two frequencies \( f_1 = 0.132U_c/\delta_{\omega f} \) and \( f_2 = f_1/2 \) (where \( \delta_{\omega f} \) is the local vorticity thickness at the forcing location and \( U_c = (U_1 + U_2)/2 \) is the vortex convection velocity) with a small amplitude incompressible excitation located just downstream of the inlet as performed by [5].

For more information about the numerical techniques, see [28, 27] where
the additional treatments ensuring silent boundary conditions (buffer zone with aerodynamic and acoustic dissipation/absorption), inlet conditions and near-inflow forcing are presented in detail and carefully validated in the present context of direct sound computation from a spatial mixing layer.

3 Isothermal case

As shown in figure 2, for the low Mach numbers considered here, no significant change in the dynamic features of the flow can be noticed. As already observed by [2] for the direct computation of sound by an axisymmetric round jets, the vortex roll-up and pairing locations only move slightly further downstream as the Mach number is increased. This behaviour is consistent with the predictions of linear stability concerning the compressibility effects, with a reduction of the disturbance growth rate as the Mach number is increased [30].

Concerning the acoustic field, Mach number effects are found to be more significant, as shown in figure 3 that presents the dilation field in the source region and in the far acoustic field. In agreement with [12, 5], the visual source
location seems to correspond to the region where the vortex pairing occurs. Because this general behaviour is recovered here for the five Mach numbers considered, the role of vortex pairing as the dominant sound source is confirmed as far as the present Mach number range is concerned. As expected, the acoustic wavelengths are gradually reduced as the Mach number is increased, the source region (of virtually unchanged size) becoming acoustically less compact in these conditions.

In addition to the change of acoustic wavelength, figure 3 also reveals a modification of the wavefront shape depending on the Mach number. Wavefronts are gradually expanded in the downstream direction and compressed upstream as the Mach number is increased, passing from a near-circular shape to an asymmetric oval shape. This wavefront change in shape and symmetry will be investigated in section 6. Note that despite the moderate Reynolds number considered here by DNS as in [12], a good qualitative agreement is found with previous studies carried out at higher Reynolds number using Large Eddy Simulation (LES) [3, 5].

The comparison among the dilatation fields obtained at each Mach number (see again figure 3 where the same colour range has been used for the five maps) clearly exhibits the increase of the acoustic intensity with the Mach number (as it can be expected) and more interestingly, a modification of the directivity of the sound emission. As already mentioned, the careful comparison between the vortex pairing obtained at each Mach number does not reveal significant differences in terms of dynamics. Then, it can be concluded that the considerable increase in the fraction of total sound radiated by the mixing layer at the highest Mach numbers, as well as the modification of its directivity properties, cannot be attributed to an obvious change of the vortex pairing process itself.

To investigate more quantitatively the modification of the acoustic field pattern by the Mach number, a diagram of the sound directivity is presented in figure 4. A set of dilatation fields like in figure 3 has been used to compute a time averaged (over a pairing cycle) root-mean square of the divergence along a circle of radius $r = 150$. To be meaningful, the circle centre should correspond
to the source location. Here, we have estimated this position (given by \( x = x_s \) and \( y = 0 \)) by simply drawing rays perpendicularly from wavefronts where the sound emission is the stronger. Depending on the (high or low velocity) streams and on the Mach number, the apparent source locations can be found approximately in the range \( x_s \in [200, 250] \), that is in the vortex paring area. A slight downstream shift of \( x_s \) is observed at the highest Mach numbers, consistently with the stabilizing influence of compressibility. However, due to the inability of the present method to locate unambiguously the acoustic source, this trend cannot be accurately discussed.

The values obtained for each \( x_s \) has been used to define each circle leading to the directivity diagram presented in figure 4 (left). Note that the use of a common value for all the circles (with for instance \( x_s \approx 225 \)) would not change significantly the shape of this diagram. For all the Mach numbers considered, a well marked lobe of directivity can be noticed in figure 4 for each side of the mixing layer, a pattern already described by [5] but not observed by [3] who reported a slightly biased directivity with two lobes in the lower and upper regions. The main influence of the Mach number concerns the sound directivity with a progressive reorientation of the acoustic radiation toward the downstream direction. Denoting the streamwise direction by \( \theta = 0^\circ \), the radiation angle \( \theta \) is found to decrease from \( \theta = 64^\circ \) at \( M = 0.2 \) to \( \theta = 42^\circ \) at \( M = 0.4 \) in the high-speed region of the mixing layer. In the low-speed region, the influence of the Mach number is less marked with \( \theta = -59^\circ \) at \( M = 0.2 \) against \( \theta = -52^\circ \) at \( M = 0.4 \).

The reinforcement of directivity combined with the sound peaks toward shallow radiation angle can be confirmed straightforwardly through the computation of the sound intensity field defined as

\[
I = \frac{(p - \bar{p}_o)^2}{\rho^2 c^4}.
\]

where \( p \) is the pressure and \( \bar{p}_o \) is a local space time-averaged pressure. Two examples of the sound intensity field are presented in figure 5 for \( M = 0.2 \) and \( M = 0.4 \). For the low Mach number case, \( I \) is found to spread over wide regions of the acoustic field in both sides of the mixing layer without well-defined directions along which the sound radiation is more efficient. Conversely, at \( M = 0.4 \), the sound intensity pattern reveals clearly a preferential direction of sound radiation inside each stream of the mixing layer at shallow angle. Note that the two visualizations presented in figure 5 are based on two different colormaps to account for the two orders of magnitude increase of \( I \)-levels from \( M = 0.2 \) to \( M = 0.4 \). The corresponding Mach number scaling of \( I \) will be discussed in the following.

In figure 6 are represented radial profiles obtained from figure 5 in the lower stream for \( M = 0.2 \) and \( M = 0.4 \) using again the apparent source location
Figure 4: Sound directivity patterns of isothermal mixing layers at Mach numbers range: $0.2 \leq M \leq 0.4$, computed as the time-averaged root-mean square of the divergence in semi-circles of radius $r = 150$, centred on the apparent source location $x_s$. Left: isothermal case. Right: non-isothermal case.

Figure 5: Sound intensity patterns of the isothermal mixing layer at Mach numbers $M = 0.2$ and $0.4$. Full computational domain including the buffer zone.
to define the origin. In both mixing layers, two distinct regions can be clearly identified: the high-amplitude nonlinear pressure fluctuations in the near-field located at small source distances \((r < 40)\) and the low-amplitude pressure fluctuations in the far acoustic field \((r > 100)\). The transition between these two regions can be detected through a local minimum of \(I\) at shallow radiation angle for the two Mach numbers presented. This very local reduction of \(I\) can be interpreted as the signature of flow-acoustic interactions occurring at the mixing-layer boundary through interference mechanisms studied experimentally by [10] in the neighbourhood of a turbulent jet. In the present mixing layer where only two different frequencies are involved, this interference process can be expected as very efficient to cancel noise emission in particular directions.

In agreement with [2] for a low Mach number axisymmetric jet, the disparity of energy scales is especially acute in the mixing layer at \(M = 0.2\) (see figure 6, top) where the sound intensity is found approximately \(10^4\) smaller in the far-field compared with the near-field. Additionally, a decaying of \(I\) following the expected law \(I \propto r^{-1}\) is recovered in this case for all the significant radiation angles. Conversely, at \(M = 0.4\), the decay of the sound intensity with source distance becomes strongly dependent on the radiation angle. The behaviour \(I \propto r^{-1}\) is only observed at the shallow radiation angle corresponding to the maximal sound radiation for this Mach number. Outside this specific angle, the \(I\) decay is found to be drastically stronger, confirming again the more directive character of sound emission as the Mach number is increased.

### 4 Non-isothermal case

Here, we consider results obtained in the non-isothermal case with the temperature ratio \(T_1/T_2 = 2\) so that the lowest stream of the mixing layer (of velocity \(U_2\)) is also the coldest, as for the case of a hot jet in a colder surrounding fluid. Dynamically, the thermal influence can be expected through baroclinic effects. Figure 7 presents a comparison between the vorticity field obtained in the isothermal (top) and non-isothermal (bottom) cases for \(M = 0.25\). In this figure, it can be observed that the vortex roll-up and pairing occur slightly earlier (i.e. more upstream) in the non-isothermal case. Additionally, the vorticity levels are found slightly intensified by the temperature ratio with the presence of counter-rotating vorticity (of opposite sign with respect to the main shear) as already observed by previous authors [29, 16, 19]. At the low Mach numbers considered, no significant counter-rotating vorticity can be observed in the isothermal case. The counter-rotating vorticity created in the non-isothermal cases is the signature of baroclinic effects through a physical mechanism that can be understood in terms of potential temperature conservation [23]. Despite these slight dynamical differences between present iso- and non-isothermal mix-
Figure 6: Logarithmic representation of the near- and far-field sound intensity radiated in the lower stream of isothermal mixing layers at Mach numbers $M = 0.2$ and $0.4$.

ing layer, because vortex roll-up and pairing seem to be triggered similarly in both cases (at comparable location), a direct comparison between the sound emission obtained in each case can be addressed relevantly.

A comparison between the acoustic field obtained for the iso- and non-isothermal mixing layers at the same Mach number ($M = 0.25$) is presented in figure 8. In the isothermal case (left), the sound propagation is the same in both streams. This symmetry leads to wavefronts with similar wavelengths and positions at the top and the bottom of the mixing layer, with only slight differences due to convection effects that are expected moderate for this low Mach number case. In the non-isothermal case (right), the sound propagates faster in the hot region so that larger acoustic wavelengths can be clearly observed, as expected. More precisely, we have checked that the main sound frequency is the same in the two sides of the mixing layer while being again tuned to the vortex pairing frequency $f_2$ as in the isothermal case. The resulting sound field pattern is found to change its shape according to the local sound speed ($c_1$ or $c_2$ with $c_1/c_2 = \sqrt{2}$), the acoustic wavelength $\lambda_i$ (with $i = 1, 2$) following the simple relation

$$\lambda_i = \frac{U_i \cos \theta + c_i}{f_2}$$

(4)
Figure 7: Vorticity fields of the isothermal (top) and non-isothermal (bottom) mixing layers at $M = 0.25$. Vorticity colored in the interval $[-1.1 : 0.2]$. Positive values (counter-rotating vorticity) are indicated in yellow (buffer zone hidden for clarity).

that takes convection effects (with the presence of $U_i$) into account depending on the radiation angle $\theta$ considered. An equivalent agreement is found for the other Mach numbers presented in this paper.

Concerning the compressibility effects, as in the isothermal case, the quite narrow range of Mach numbers considered here does not allow the observation of significant dynamical effects, with comparable primary and secondary instabilities that are slightly delayed as expected. Figure 9 exhibits clearly the similarity between each flow, with no discernable modifications of baroclinic effects (counter-rotating vorticity) depending on the Mach number.

As in the isothermal case, the drastic influence of the Mach number increase on the acoustic emission is recovered here, with a reinforcement of the sound levels and directivity. Note however that these trends, illustrated in figure 3, are less marked in the hot side of the mixing layer. The minor influence of the Mach number in the hot region can be attributed to the larger acoustic wavelengths created in this area (due to the higher sound speed) that make the source zone more compact and then less acoustically directive by comparison with the cold region. This preliminary conclusion can be quantitatively confirmed by the examination of the sound directivity pattern generated as for the isothermal case and presented in figure 4 (right). For the non-isothermal case, secondary lobes can be observed upstream from the apparent source location, these lobes being not present in the isothermal case. Concerning the dominant lobes in the non-isothermal case, they clearly do not increase in magnitude at the same rate as the Mach number is increased. The reduction of the compactness of the acoustic sources in the hot region seems to moderate the rise of the sound emission as the Mach number is increased, resulting in small lobes in the upper stream of the mixing layer.
Figure 8: Dilatation field of the isothermal (left) and non-isothermal (right) mixing layers at Mach number 0.25 (buffer zone hidden for clarity). Dilatation colored in the interval $[-10^{-4} : 10^{-4}]$.

Figure 9: Vorticity field of non-isothermal mixing layers at Mach numbers $M = 0.2, 0.25, 0.3, 0.35$ and 0.4 (from top to bottom). Vorticity coloured in the interval $[-1.1 : 0.2]$. Positive iso-contour levels (counter-rotating vorticity) are indicated in yellow. Representation of the near-field mixing region $0 \leq x \leq 350$ and $-15 \leq y \leq 15$. 
5 Mach number scaling

Mach number effects can be considered through the scaling of the acoustic intensity (3) presented in figure 11. For the isothermal case, $I_{\text{max}}$ is found to scale as $M^7$ in the two sides of the mixing layer. This behaviour is consistent with the theoretical analysis of [20]. In this work, the power radiated by low Mach number two-dimensional turbulence is shown to be proportional to $\ell \rho_o v^3 M^4$ where $\ell$ and $v$ are the characteristic size and velocity of individual sources (eddies) respectively while $\rho_o$ is the reference density. In the present scaling, this dimensional analysis leads to a $M^7$ law.

Concerning the non-isothermal case, the $M^7$ law is only recovered in the lower stream. On the contrary, the acoustic intensity in the high-speed region does not follow any power law. This behaviour suggests a change in the nature of the acoustic sources inside this Mach number range with a significant role played by the entropic sources as shown by [16, 19] for a temporal mixing layer.

Comparison between the acoustic intensities obtained from the isothermal and non-isothermal mixing layers shows that the former is noisier than the latter in both sides. This trend is in contradiction with experimental observations of [15] where a hot jet is found to be noisier than a cold jet at low Mach number. The inability of a 2D spatial mixing layer to reproduce this behaviour suggests that this simple flow configuration cannot be assimilated as a good reduced model to capture fundamental features of hot jet noise emission.

6 Analysis of the acoustic wavefront pattern

In this section, we are interested in interpreting the sharpening of the directivity peak as the Mach number increases, as observed in figures 3 and 4 (a). This trend could be considered through the framework introduced by [33], which models the influence of a turbulent interface (shear layer) between a uniform flow and a
Figure 11: Maximum far-field sound intensity ($I_{\text{max}}$) radiated by isothermal and non-isothermal mixing layers in the Mach numbers range $0.2 \leq M \leq 0.4$ at the distance $r = 300$ from the apparent source location.

In the following, three additional frameworks are considered: the compactness of a wavepacket, the acoustic analogy, and the ray theory. The analysis, restricted to the isothermal case for simplicity, is performed for two different
Mach numbers $M = 0.25$ and $0.4$ with the aim to examine possible changes in the acoustic source structure or propagation medium while trying to establish connections between source contributions and modification of the acoustic field.

Near-field pressure wave packet. If we consider the instantaneous pressure at the intermediate location where hydrodynamic fluctuations vanish while acoustic waves start to emerge, namely $y \approx 20$, it is interesting to observe a pattern similar to a wave packet as shown in figure 12. The contraction of the packet amplitude in the vicinity of $x/\delta_\omega = 200$ is noteworthy. The influence of the form of the envelope curve of a pressure wave packet on the directivity was investigated by previous authors [13, 1]. Figure 12 shows that in the present mixing-layer, the form of the envelope curve remains almost the same at the two Mach numbers. Consequently, the enhancement of the directivity observed at $M = 0.4$ cannot be explained by such effect in that case.

Acoustic analogy. Another way to interpret the increase of sound directivity with the Mach number could be to observe a possible change of shape of the source term. To illustrate this possibility, figure 13 (left) compares the spatial distribution of the rms value of the fluctuating Lighthill source term $S'$ with $S = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$ for the two Mach numbers considered ($M = 0.25, 0.4$). The two patterns obtained are found to be very close, confirming that compressible effects are weak in the dynamically active region of the flow. It is also worth noting
that the source maximum is not located at the apparent source location but significantly upstream, near \( x \approx 100 \). This type of view does not give any clue about the reason of the strong modification of the sound waves observed in the acoustic field. However, if the same data are observed through a normalization based on acoustic wavelength scales (figure 13, right), the change of the source size can be directly observed. This loss of compacity as the Mach number is increased is certainly an important feature influencing the sound directivity (see for instance the discussion in [21, 9]), but how this change modifies the acoustic wave propagation remains to be clarified.

For that purpose, the integral solution of the Lighthill equation

\[
\frac{1}{c_2^2} \frac{\partial^2 p_a}{\partial t^2} - \frac{\partial^2 p_a}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

may be computed with an accurate approximation of the Lighthill tensor taken as

\[
T_{ij} = \rho u_i u_j
\]

once it has been checked that both the entropic and viscous terms have a negligible contribution to the acoustic field for the isothermal flows considered in this section.

Using the Green function formalism, it is easy to express the integral solution of equation (5) but its computation is not an easy task. In practice, it requires to deal with different times from the source region towards the observer zone where the acoustic prediction is performed. Using an efficient iterative algorithm, the computation of the integral solution could be carried out as in [26] but with
the same intrinsic limitation connected to the use of a 3D Green function that prevents quantitative comparisons between the directly computed sound and its prediction solving (5). To allow the accurate estimation of each specific acoustic source contribution, the use of a 2D Green function combined with the same algorithm is required for the present 2D flow configuration. Unfortunately, this would be too much computationally demanding due to the particular form of the 2D Green function in the time domain. It requires to consider the full history of the flow throughout the calculation, even for the single computation of one acoustic pressure snapshot.

To overcome this difficulty, we can here take advantage of the time periodicity of the flow imposed by the inflow forcing based only on two discrete frequencies $f_1$ and $f_2$ with $f_2 = f_1/2$. Furthermore, as previously discussed, the examination of the acoustic signal in the far field shows a strongly dominant frequency $f_2$ associated with the pairing. In this particular context, the solving of equation (5) in the frequency domain can be limited to a single frequency $f_2$ without losing any significant contribution to the full acoustic emissions. The corresponding solution can be written as

$$\hat{p}_a(x, f_2) = \int_D \hat{G}(x|y, f_2) \frac{\partial^2 \hat{\Gamma}_{ij}(y, f_2)}{\partial y_i \partial y_j} dy$$

where the Green function in the Fourier space is given by

$$\hat{G}(x|y, f_2) = i \frac{4}{4} H_0^{(2)} \left( \frac{2\pi f_2 |x - y|}{c_2} \right)$$

and $H_0^{(2)}$ is the Hankel function of second type. Provided that $D$ covers the whole DNS domain, convection effects are included in the Lighthill source term, and the integral solution is correctly obtained using the Green function for a uniform medium at rest as illustrated by [6,26]. Provided that $D$ covers the whole DNS domain, that the observer is located inside it, and that the source data come from the DNS itself, convection effects are included in the Lighthill source term, and the integral solution is correctly obtained using the Green function for a uniform medium at rest as illustrated by [6, 26].

Using the instantaneous data collected over a single cycle $T = 1/f_2$, the acoustic pressure predicted using (7) has been computed on a regular Cartesian grid $150 \times 150$ covering the computational domain at any instant with

$$p_a(x, t) = \hat{p}_a(x, f_2) \exp(i2\pi f_2 t).$$

The resulting maps are presented in figure 14 by comparison with the data computed directly by DNS at the same arbitrary instant. The excellent agreement observed at the two Mach numbers $M = 0.25$ and 0.4 validates both the numerical procedure to perform the analogy and the assumption about the purely
Figure 14: Comparison between the acoustic pressure $p_a$ computed directly by DNS (left) or using the acoustic analogy (right) for two Mach numbers $M = 0.25$ (bottom) and $M = 0.4$ (top) in the isothermal case.
The harmonic character of the noise emission at the frequency $f_2$. A more detailed comparison is also shown in figure 15 where the curve of the predicted acoustic pressure is found to fit very closely its direct DNS counterpart with an overall amplitude decrease of $r^{-1/2}$ as expected for a 2D flow configuration [20]. The most remarkable feature is that the acoustic analogy is found to capture accurately the main trends of the acoustic waves associated with the Mach number, namely their relevant change of wavelength, amplitude and the resulting sound directivity. Based on this favourable agreement, it seems appropriate to examine each component of the Lighthill tensor in order to determine if these changes can be connected to a modification of the balance between specific terms for a given decomposition.

To distinguish the contribution of velocity from density fluctuations on the noise emission, we consider here the Lighthill tensor in the framework of the Reynolds decomposition of the velocity $u_i = \bar{u}_i + u'_i$ and the density $\rho = \bar{\rho} + \rho'$ that can be expressed as

$$T_{ij} = \bar{\rho}_i \bar{u}_j + \bar{\rho}(\bar{u}_i u'_j + u'_i \bar{u}_j) + \rho' \bar{u}_i \bar{u}_j + \rho'(\bar{u}_i u'_j + u'_i \bar{u}_j) + \bar{\rho} u'_i u'_j + \rho' u'_i u'_j$$  \hspace{1cm} \text{(10)}$$

Here, due to the time periodicity of the flow, the overbar operator refers to a time average over a single cycle duration $T = 1/f_2$ whereas the prime denotes the deviation of the instantaneous quantity from its average. We use here the same notation as [4] where $L$, $Q$ and $C$ refers to the linear, quadratic or cubic character of the terms with respect to the velocity and density fluctuations.
Figure 16: Diagrams of acoustic intensity at the radius $r = 150$ for two Mach numbers $M = 0.25$ (left) and $M = 0.4$ (right) in the isothermal case. Direct data (DNS) compared with acoustic analogy predictions $\Sigma_5$ (as the sum of the 5 individual terms from equation (10)) and with separate contributions from $L_2$ or $L_1 + Q_1 + Q_2 + C$.

Because the mean term $M$ is stationary, it has no contribution in the noise emission and can be ignored in the analysis. Using a Reynolds decomposition only on the velocity, some previous authors [18, 24] have also distinguished the self-noise, associated with $Q_2 + C = \rho u_i' u_j'$, from the shear noise, associated with $Q_1 + L_1 = \rho (\bar{u}_i u_j' + u_i' \bar{u}_j)$. In this simpler decomposition, the residual term $M + L_2 = \rho \bar{u}_i \bar{u}_j$ (that is exactly identical to $L_2$ in terms of noise emission) is not expected to be of major importance in the sense that the single source of unsteadiness is related to density fluctuations that are a priori weak at low compressibility. In what follows, it will be seen that this a priori conclusion needs to be qualified, even for a moderate Mach number flow.

As in any decomposition, the physical analysis of each term based on (10) is not necessarily meaningful by emphasizing some cancellation effects between terms that are individually very much noisier than the full source term (see for instance [7]). On the contrary, it can be enlightening to identify negligible contributions while trying to associate them with a physical feature. Potentially instructive information can also be detected through the presence of a significant contribution that was expected to be of minor importance. Based on the present results, the contribution of each term of the decomposition (10) has been examined. It has been found in particular that the cubic and quadratic terms $C$ and $Q_1$ were negligible with a dominant influence of $Q_2$ and $\bar{\rho} \bar{u}_i u_j'$ in $L_1$.

In the present analysis, the most remarkable feature concerns $L_2$ through
the contribution of \( \rho' \bar{u}_1 \bar{u}_1 \) (the two other terms \( \rho' \bar{u}_2 \bar{u}_2 \) and \( \rho' \bar{u}_1 \bar{u}_2 \) are negligible due to the weak mean transverse velocity). This term is the single one that involves only the contribution of density fluctuations on the noise emission. For the present low Mach number cases where compressible effects are very weak, as already stated, \( L_2 \) is expected to be weak if not negligible. For \( M = 0.25 \), this minor role of \( L_2 \) is confirmed as shown in figure 16 (left) where the acoustic intensity is found mainly governed by the sum \( L_1 + Q_1 + Q_2 + C \) except near the specific direction \( \theta \approx 45^\circ \). However, the same cannot be said for \( M = 0.4 \) where the increase of the directivity is clearly connected to the increase of the role of density fluctuations in the noise emission process, as shown in figure 16 (right) where the reinforcement of the acoustic intensity at shallow angles can be associated with the increase of the \( L_2 \) influence. To our knowledge, this is the first time that a significant role of density fluctuations in the source term is observed in a so weakly compressible flow. As a first consequence, in acoustic modelling based on the shear- and self-noise distinction, the residual term should not be ignored. In physical terms, the role of \( L_2 \) observed here suggests that density fluctuations play an important role in the increase of the directivity with the Mach number. These fluctuations become very efficient in the noise generation process despite their low level in the present context of weakly compressible and isothermal flow considered here.

**Acoustic propagation in moving media.** Finally, the acoustic wavefront deformation due to a radiation in a moving medium is investigated. First, the convection effect on the acoustic propagation is illustrated in figure 17, through the sound field of a harmonic monopole located on the centreline at \( x_s = 200 \) and emitting toward the different streams considered here. Those fields are computed using the convected Green function in the Fourier space given by [25] with

\[
\hat{G}_c(x|y, f_2) = \frac{i}{4\beta} \exp \left( \frac{2i\pi M_i f_2 (x_1 - y_1)}{c_2 \beta^2} \right) H^{(2)}_0 \left( \frac{2\pi f_2 r_\beta}{c_2 \beta^2} \right)
\]

where \( M_i \) is the moving medium Mach number (\( i = 1, 2 \)), \( \beta^2 = 1 - M_i^2 \), \( r_\beta = \sqrt{(x_1 - y_1)^2 - \beta^2(x_2 - y_2)} \), \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \). This visualisation exhibits the wavelength deformation very well, as visible in figure 17 (left) where the simple scaling given by equation (4) is also well illustrated. However, such convection effect has strictly no influence on the wave amplitude at a constant distance from the source, as visible on the corresponding acoustic wave longitudinal profiles plotted in figure 17 (right). Consequently, effects of the wave convection may not explain the deviation of the directivity peak observed in the present DNS as the Mach number is increased.

More relevant seem to be the ray trajectories through the mixing layer from a point source located in the same location as before. For this computation,
Figure 17: Acoustic wave propagation emitted by a harmonic monopole located at $x_s = 200$ in a moving medium with Mach numbers $M_i$ corresponding to the high- and low-speed free stream velocities of the two cases $M = 0.25, 0.4$. Left: maps of acoustic pressure $p_a$. Right: profiles of acoustic pressure $p_a$ at $y = 0$. 
the velocity field used is a hyperbolic tangent profile with similar thickness as the mean flow around the source location. Ray tracing is a simple model to visualize refraction effects, the latter being extensively investigated by [32] through a Green function derivation. Naturally, the ray theory assumption about the acoustic wavelength restriction may not apply in the present shear flow where the wavelength associated with $f_2$ rises to several times the extent of the mixing layer thickness, consistently with the assumption used in [33] as discussed at the beginning of this section. Strictly speaking, ray trajectories do not give access to directivity as a trace of wave amplitude. But they are able to reveal the creation of quiet zone or screen effect due to the wave refraction. In that sense, the agreement between this ray pattern deviation when the Mach number increases and the pressure pattern from the DNS or analogy results is noteworthy as shown in figure 18. The influence of the Mach number on the shape of the acoustic field appears here through refraction effect, and this may be considered as an interesting interpretation of the directivity change observed in the present DNS results.

7 Conclusion

The direct computation of sound from spatial mixing layer has been carried out at five different Mach numbers ranging from 0.2 to 0.4 while considering thermal effects. As governing equations of compressible flow dynamics, the characteristic-based formulation of [31] has been used, allowing to implement more easily silent boundary conditions (i.e. without generation of spurious noise), a property difficult to reach for the low Mach number range considered. In the isothermal case, all the trends observed are consistent with the previous results of [12, 5] who have considered a very close computational configuration. The aim of this work was to consider the effects of Mach number and temperature while focusing on low compressible regimes where the acoustic sources are expected to be weakly modified.

In agreement with previous studies, the vortex paring process (that is controlled through a two-harmonic forcing) is found to be the dominant sound source, this trend being confirmed for all the Mach numbers considered here. In terms of compressible effects on the dynamics, vortex roll-up and pairing are found to be slightly delayed (i.e. moved further downstream) as the Mach number is increased. On the contrary, Mach number effects are found to modify strongly the acoustic emissions through the expected increase of the noise ($M^7$ scaling of the acoustic intensity) but also through the reinforcement of the sound directivity as the Mach number is increased. This behaviour can be associated with the reduction of the compactness of the acoustic source that makes the sound emission more directive with shallower radiation angles as the Mach
Figure 18: Ray trajectories from a point location at $x_s = 200$. 

$M_1 = 0.5, M_2 = 0.25$

$M_1 = 0.8, M_2 = 0.4$
number is increased. Another interesting feature is the strong increase of the density fluctuations contribution in the acoustic sources for these shallow angles, as shown by the analysis based on the acoustic analogy. The efficiency of this process is unexpected regarding the low amplitude of the density fluctuations for the weakly compressible and isothermal cases considered here. In terms of acoustic source modelling, the contribution of density fluctuations should not be neglected, even at moderate low Mach number where compressible effects cannot have significant dynamical effects. As a consequence, the common decomposition in shear- and self-noise can be seen as uncompleted if the density fluctuations are not explicitly taken into account. Naturally, the importance of density fluctuations cannot give a complete picture of the mechanisms responsible of the sound directivity, this change of influence being found to combine with convection and refraction effects to shape the acoustic wavefront pattern depending on the wave Mach number. However, the refraction effect may be taken with care, regarding the respective value of the wavelength and the layer thickness. In a similar context, Ffowcs-Williams modelling [33] leads to the conclusion that there should be no shear effect on the propagation.

Dynamically, the main non-isothermal effects consist in the intensification of vorticity levels with the production of counter-rotating vorticity, leading to a more complex pattern of the acoustic sources. Acoustically, this change of the source structure is found to lead to a reduction of sound emission. In the upper/hot stream, the Mach number scaling of the acoustic intensity does not exhibit any power law, confirming the difficulty to predict the sound emission from non-isothermal flows.

Due to the high computational cost of present DNS based on high-order schemes on a fine grid, the present parametric study is restricted to 2D computations at low Reynolds number. The resulting flow is an interesting reduced model even if it cannot reproduce the non-isothermal features of jet noise. To expect a quantitative agreement with real-life flow, 3D effects must be taken into account, especially for considering a wider range of Mach number. The same parametric study should be soon affordable in 3D using the new generation of massively parallel supercomputers, providing a reliable database to improve the various strategies of sound prediction based on an acoustic analogy. In a second step, more realistic values for the Reynolds number should be addressed. Regarding the too much demanding computational cost ($C_C$) of DNS with respect to an increase of the Reynolds number ($Re$), through the approximate scaling $C_C \propto Re^3$, the extension of the present investigation to high-$Re$ flows will require, at least in the next decade, the use of Large Eddy Simulation (LES) for the computation of the acoustic sources. The combination of reliable databases from 2D/3D DNS (as reduced model) and 3D LES (as realistic model) will probably be a fruitful way to improve the various strategies of sound prediction based on an acoustic analogy.
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References


