Static and dynamic analysis of liquid and gas annular seals

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University of Poitiers: 25000 students on three campuses

Institute P »: 270 permanent employees and as many PhD students, National Research Council’s second largest institute
Cryogenic turbopump

I. Static problem: \( \Delta P = \Delta P(\dot{M}) \)

- pressure difference: \( \Delta P = P_{\text{inlet}}^0 - P_{\text{exit}}^0 \)
- mass flow rate: \( \dot{M} = \rho W_m 2\pi RC \)

II. Dynamic problem:

\[
- \begin{bmatrix} F_{x1} \\ F_{y1} \end{bmatrix} = \begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix} \begin{bmatrix} e_{x1} \\ e_{y1} \end{bmatrix} + \begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix} \begin{bmatrix} \dot{e}_{x1} \\ \dot{e}_{y1} \end{bmatrix} + \begin{bmatrix} M_{XX} & M_{XY} \\ M_{YX} & M_{YY} \end{bmatrix} \begin{bmatrix} \ddot{e}_{x1} \\ \ddot{e}_{y1} \end{bmatrix}
\]

I. Static problem for straight annular seals: analytic solution

$$P_{inlet}^0 - P_{exit}^0 = (1 + \xi_{inlet}) \frac{\rho W_m^2}{2} + \frac{\lambda L}{2C} \frac{\rho W_m^2}{2} - (1 - \xi_{exit}) \frac{\rho W_m^2}{2}$$

**Main problem: the definition of the friction factor**

$$\lambda = \lambda(Re, r, \ldots), \quad Re = \frac{\rho W_m^2 C}{\mu}$$
The definition of the friction factor

\[ \lambda_{\text{Blasius}} = 0.3164 Re_m^{-0.25}, \quad Re_m = \frac{\rho W_m 2C}{\mu} \]

\[ \lambda_{\text{Yamada}} = \frac{0.3164 Re_{ref}^{-0.25}}{W_m/V_{ref}}, \quad Re_{ref} = \frac{\rho V_{ref} 2C}{\mu} \]

\[ V_{ref} = \sqrt{W_m^2 + U_m^2}, \quad U_m = \gamma R \Omega \]

\[ f_{\text{Moody}} = \frac{\lambda}{4} = 1.375 \cdot 10^{-3} \left[ 1 + \left( \frac{10^4 r}{C} + \frac{10^6}{Re_{ref}} \right)^{1/3} \right] \]

\( r \): roughness height \((r<<C)\)
For straight annular seals the « analytic » solution is a first approximation if:
1. the circumferential velocity is neglected
2. the rotor is centered

**Numerical solutions:**

1. Resolution of the full Navier-Stokes equations: Computational Fluid Dynamics
2. Lubrication: simplified **thin film** flow model dominated by **inertia**

The « bulk flow » equations, $Re^* = Re.C/R >> 1$

$$\frac{\partial (\rho H)}{\partial t} + \frac{\partial (\rho HW)}{\partial z} + \frac{\partial (\rho HW)}{\partial x} = 0$$

$$\frac{\partial (\rho HW)}{\partial t} + \frac{\partial (\rho HWU)}{\partial z} + \frac{\partial (\rho HWU)}{\partial x} = -H \frac{\partial p}{\partial z} + \tau_{Sz} + \tau_{Rz}$$

$$\frac{\partial (\rho HU)}{\partial t} + \frac{\partial (\rho HWU)}{\partial z} + \frac{\partial (\rho HUU)}{\partial x} = -H \frac{\partial p}{\partial x} + \tau_{Sx} + \tau_{Rx}$$

Main results:
- The rotor velocity will decrease the flow rate
- The rotor eccentricity will increase the flow rate
How to improve the seal efficiency of the annular seal?

By using *grooves* and *steps*: **labyrinth seals**

Are there any analytic models available? 
NO

What models can be used? 
- Simplified models (extension of the « bulk flow » model) 
- Computational Fluid Dynamics
Simplified models for stator-grooved seals (extension of the « bulk flow » model)

Pressure isolines

Pressure isolines
Computational Fluid Dynamics:

Mesh generation
- Initial coarse mesh
- Intermediate mesh
- Final (refined) mesh

Obtaining convergence

Interpreting the results

Commercial codes: FLUENT, CFX, FloWorks…
II. Dynamic problem for annular seals

\[-\{F_{x1}\} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \{e_{x1}\} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \{\dot{e}_{x1}\} + \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \{\ddot{e}_{x1}\}\]

- Analytic model of the flow in a straight annular seal

Superposition of three effects:

- The Lomakin force (restoring effect)
- The inertia force (Bernoulli effect)
- The viscous force (« oil wedge » effect)
The Lomakin force, $F_L$ (restoring effect)
The inertia force $F_I$

Bernoulli equation:
$P + \rho U_m^2/2 = \text{const.}$

continuity equation:
$\rho U_m H = \text{const.}$

The viscous force
(« oil wedge » effect)
The Lomakin force (restoring effect)

\[ F_L \]

The inertia forces
Bernoulli effect

\[ F_I \]

The viscous force (« wedge effect »)

\[ F_V \]

\[ F_X = F_L - F_I \]

\[ F_Y = k_3 W_m \frac{\rho R \Omega}{U_m} \]

\[ k_1 = \frac{3\pi}{2} f_{\lambda_o} \frac{\rho R^3}{C} T(L/R) \epsilon \]

\[ k_2 = \pi \rho L R T(L/R) \epsilon \]

\[ k_3 = \frac{\pi}{4} f_{\nu_o} \rho R^2 T(L/R) \epsilon \]

\[ T(L/R) = 1 - \frac{\tanh(L/2R)}{L/2R} \]
\[ \omega: \text{whirl velocity around the centered position} \]

\[ \begin{align*}
\begin{bmatrix}
F_X \\
F_Y
\end{bmatrix}
&= 
\begin{bmatrix}
K & k & e_x \\
-k & K & e_y \\
C & c & \dot{e}_x \\
-c & C & \dot{e}_y \\
M & 0 & \ddot{e}_x \\
0 & M & \ddot{e}_y
\end{bmatrix}
\begin{bmatrix}
e_X \\
e_Y \\
\dot{e}_x \\
\dot{e}_y \\
\ddot{e}_x \\
\ddot{e}_y
\end{bmatrix}
\]

\[ \begin{align*}
F_r &= -k_1 W_m^2 + k_2 (\mathcal{F}_R \Omega - R \omega)^2 \\
F_t &= k_3 W_m (\mathcal{F}_R \Omega - R \omega)^\gamma_m \\
\frac{F_r}{\varepsilon C} &= -K - c \omega + M \omega^2 \\
\frac{F_t}{\varepsilon C} &= k - C \omega
\end{align*} \]

\[ K = \frac{k_1 W_m^2 - k_2 U_m^2}{\varepsilon C}, \quad \text{Lomakin – inertia (Bernoulli)} \]

\[ k = \frac{k_3 W_m U_m}{\varepsilon C}, \quad \text{viscous ("oil wedge")} \]

\[ C = \frac{k_3 W_m R}{\varepsilon C}, \quad \text{viscous ("squeeze")} \]

\[ c = \frac{2k_2 \mathcal{F}_R^2 \Omega}{\varepsilon C}, \quad \text{inertia (Bernoulli)} \]

\[ M = \frac{k_2 R^2}{\varepsilon C}, \quad \text{inertia (Bernoulli)} \]
1. Can we accurately and efficiently calculate straight annular seals?

YES, if:
- The inlet pressure drop effect,
- The exit recovery effect,
- The prerotation velocity,

are correctly estimated. How can these effects be estimated?

- Informed guess or parametric study,
- Measurements,
- Computational Fluid Dynamics (CFD).
2. Can grooved or stepped annular seals be calculated accurately and efficiently?

-The same problems as for straight annular seals (inlet - exit pressure drop and prerotation)

-Grooved seals can be calculated efficiently by using simplified methods (extended « bulk flow » methods). These methods are efficient but not always accurate.

-There are no simplified methods for stepped seals.

-CFD is the only accurate approach but it is difficult to handle and time consuming.

3. How accurate should be the calculation model?

-As accurate as possible?

-If inlet and exit conditions cannot be accurately estimated then simplified methods are good enough and full CFD approaches are hardly justified.
Gas (air) annular seals

The same problems but compressibility must be taken into account:

$$Mach \ number, \ M = \frac{Fluid \ velocity}{Local \ sound \ speed}$$

Incompressible $\rho=const$, $M<0.3$

Compressible $\rho\neq const$, $M>1$

- Subsonic $0.3<M<0.9$
- Transonic $0.9<M<1.1$
- Supersonic $1.1<M<5$
- Highly supersonic $M>5$

<table>
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<tr>
<th>Accelerating flow</th>
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<th>Viscous flow in a parallel channel</th>
<th>Friction force</th>
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<td>Decreasing flow section</td>
<td>$U_m$</td>
<td>Friction force</td>
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$$\frac{dM^2}{dz} = \frac{2M^2}{H} \left(1 + \frac{\gamma - 1}{2} M^2\right) \frac{\gamma M^2 - \frac{dH}{dz}}{1 - M^2}$$
Choked flow: Mach number $M=1$ in the exit section of the annular seal

\[
\frac{P - P_{\text{exit}}}{P_{\text{inlet}} - P_{\text{exit}}}
\]

Dimensionless axial length

Pressure, subsonic exit
Mach, subsonic exit
Pressure, choked exit
Mach, choked exit
Lomakin effect in gas (air) annular seals: subsonic flow

\[ P_{\text{inlet}} = (1 + \xi) \frac{\rho W_m^2}{2} \]

\[ P_{\text{exit}} = \frac{\rho W_m^2}{2} \left( \frac{L}{2C*\lambda} \right) \]

Graphs showing Mach number and pressure as functions of axial distance for subsonic flow with various eccentricities.
Lomakin effect in gas (air) annular seals: choked flow and static instability
Conclusion

*Are the analysis methods good enough?*

-It depends on the accuracy needed in rotordynamic analysis.

-CFD is becoming popular (FLUENT, CFX, FloWork, …)

*Further needs:* improvement of simplified, efficient methods for damper seals (honeycombed, dimpled stator surface).
References used in this presentation:


- Amoser, M., « Stromungsfelder und Radialkrafte in Labyrinthdichtungen hydraulischer Stromungsmaschinen, Dissertation ETH Zurich, nr. 11150.
