Theoretical and experimental analysis of hybrid aerostatic bearings

Mihai ARGHIR
Professor, Fellow of the ASME
Université de Poitiers, France
Université de Poitiers: 25000 students on three campuses

Institute Pprime (P'): 270 permanent employees and as many PhD students, National Research Council’s second largest institute
Why (still) analyzing the aerostatic bearing?

OUTLINE

• Static characteristics

• Dynamic characteristics
  – Combined « bulk flow » and CFD analysis
  – Experimental analysis (validations)
  – Whirl/whip stability
  – « Pneumatic hammer » hammer instability

• Other candidates: the foil bearing

• Conclusions
Hydrostatic bearing

Supply pressure

Restrictor

Recess surface ≈ (50…70)% bearing surface
Recess depth ≈ 100 · radial clearance

Aerostatic bearing

Recess surface ≈ (10…20)% bearing surface
Recess depth < 10 · radial clearance
Static characteristics of the aerostatic bearing

The static characteristics raise no peculiar problems

Good load capacity
Responds like a linear spring up to 40…50% eccentricity with limited cross-coupling effects

The static characteristics raise no peculiar problems
Dynamic characteristics of aerostatic bearings

I. Linear rotordynamic coefficients

\[
\begin{align*}
-F_x &= -\left\{ \Delta W_x \right\} = \left[ K_{xx} \ K_{xy} \right] \left[ \Delta x \right] + \left[ C_{xx} \ C_{xy} \right] \left[ \Delta \dot{x} \right] \\
F_y &= -\left\{ \Delta W_y \right\} = \left[ K_{yx} \ K_{yy} \right] \left[ \Delta y \right] + \left[ C_{yx} \ C_{yy} \right] \left[ \Delta \dot{y} \right]
\end{align*}
\]

\[
\begin{align*}
-F_x &= -\left\{ \Delta W_x \right\} = \left[ K \ k \right] \left[ \Delta x \right] + \left[ C \ c \right] \left[ \Delta \dot{x} \right] \\
F_y &= -\left\{ \Delta W_y \right\} = \left[ -k \ K \right] \left[ \Delta y \right] + \left[ -c \ C \right] \left[ \Delta \dot{y} \right]
\end{align*}
\]

II. Stability (self-sustained vibrations)

1. Whirl/whip

\[
\begin{align*}
-F_r &= \left\{ \Delta W_r \right\} = \left[ K \ k \right] \left[ e \right] + \left[ C \ c \right] \left[ \dot{e} \right] \\
F_t &= \left\{ \Delta W_t \right\} = \left[ -k \ K \right] \left[ e \Phi \right] + \left[ -c \ C \right] \left[ \dot{e} \Phi \right]
\end{align*}
\]

2. « Pneumatic hammer »
THEORETICAL ANALYSIS

- The « bulk flow » system of equations
- CFD models
Thin film (lubrication) models

Classical lubrication assumptions for the thin film:
- The flow characteristics are considered constant over the film thickness.
- The curvature is neglected.

- Reynolds equation for compressible fluid flow

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h^3}{12 \mu} \frac{\partial P}{\partial z} \right) = \frac{\partial}{\partial x} \left( \frac{\rho U h}{2} \right) + \frac{\partial}{\partial t} (\rho h) \quad \text{inappropriate if } Re \cdot C/R > 1
\]

- The Bulk Flow system of equations (high Reynolds number):

Continuity
\[
\frac{\partial \rho h}{\partial t} + \frac{\partial \rho h W}{\partial z} + \frac{\partial \rho h U}{R \theta \theta} = 0
\]

Momentum
\[
\frac{\partial \rho h W}{\partial t} + \frac{\partial \rho h W^2}{\partial z} + \frac{\partial \rho h U W}{R \theta \theta} = -\frac{h \partial P}{\partial z} + \tau_{Sz} + \tau_{Rz} + \frac{\partial \rho h U}{\partial t} + \frac{\partial \rho h U^2}{\partial z} + \frac{\partial \rho h U W}{R \theta \theta} = -\frac{h \partial P}{R \theta \theta} + \tau_{Sz} + \tau_{Rz} + \text{an appropriate turbulence model if needed} + \text{Hir’s assumption}
\]

Energy
\[
\frac{\partial \rho i_t}{\partial t} + \frac{\partial \rho h W i_t}{\partial z} + \frac{\partial \rho h U i_t}{R \theta \theta} = q_s + q_R + R \omega \tau_{Rd}^H
\]

Eq. of state
\[
P = \rho r T \quad \text{or another equivalent thermodynamic equation}
\]

The numerical solution of the « bulk flow » system of equations is not always an easy task
Boundary conditions & local (inertia) effects

Restrictor: compressible orifice flow
\[ \dot{m}_{th} = C_d \frac{A P_s}{\sqrt{T_s}} Q_{ideal}(P_s, P_f, \gamma) \]

Recess/Film interface generalized Bernoulli equation:
\[ \frac{p_t}{p} = \left[ 1 + \frac{\gamma - 1}{2} (1 \pm \xi) M_n^2 \right]^{\frac{\gamma}{\gamma - 1}} \]

Local inertia effects cannot be predicted by thin film flow models therefore we will use CFD
What is the place of CFD (nowadays) in aerostatic bearing analysis?

A point of view:
CFD does a good job for (almost) everything; why not use it for the full size problem?

Another point of view:
Use CFD only when the thin film flow models fail:
- In the recess
- At the interface between the recess and the thin film
- In the orifice restrictor
- ...

Link CFD results to « bulk flow » analyses by using lumped parameters
**Steady CFD analysis: methodology**

1. Simplify the geometry taking into account working conditions

2. Generate the grid: for aerostatic bearings it is the hardest point

3. Solve: understand the numerical solver, tune relaxation parameters

4. Validate: don’t immediately accept any result, for example check at least for mass balance

5. Look for relevant information, i.e. results that can be cast as boundary conditions of the « bulk flow » equations: pressure patterns, rapid pressure variations, mass flow rates, etc.
Axial pressure variation stemming from CFD analysis

A constant pattern is sufficient to describe the pressure in the recess zone.
Sensitivity analysis ("bulk flow" calculations): parameters influencing the dynamic coefficients

<table>
<thead>
<tr>
<th>Parameter list</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Supply pressure</td>
</tr>
<tr>
<td>B</td>
<td>Recess pressure</td>
</tr>
<tr>
<td>C</td>
<td>Supply temperature</td>
</tr>
<tr>
<td>D</td>
<td>Clearance</td>
</tr>
<tr>
<td>E</td>
<td>Surface roughness</td>
</tr>
<tr>
<td>F</td>
<td>Pressure loss in the axial direction</td>
</tr>
<tr>
<td>G</td>
<td>Pressure loss in the circumferential direction</td>
</tr>
</tbody>
</table>

The recess pressure has a major influence on the stiffness coefficients; its value is controlled by the restrictor area and geometry.
Recess pressure and orifice modeling

Vena contracta due to the orifice restriction
\[ A_{orif} = \pi d^2 / 4 \]

Vena contracta due to the inherent restriction
\[ A_{inh} = \pi d (h + \delta) \]

\[ \dot{m}_{th} = C_d \frac{A P_s}{\sqrt{T_s}} Q_{ideal}(P_s, P_r, \gamma) \]

Which surface should be used and when?

\( \delta \): recess depth
\( h \): film thickness
The equivalent surface of the restrictor

Deep recess, $A_{orif}/A_{inh} \ll 1$

$$A_{orif} = \pi d^4 / 4$$

Shallow recess, $A_{orif}/A_{inh} \ll 1$

$$A_{inh} = \pi d(h + \delta)$$

Equivalent recess [1], $A_{orif}/A_{inh} \equiv 1$

$$A_{equiv} = \frac{A_{orif}A_{inh}}{\sqrt{A_{orif}^2 + A_{inh}^2}}$$

[1] Rieger, 1967, Design of Gas Bearings Notes, supplemental to the RPI-MTI course on gas bearing design, RPI-MTI, 2 Volumes
Mass flow rate in a deep recess
\( \frac{A_{orif}}{A_{inh}} = 0.1 \)

The restriction is ensured by the surface of the orifice

\[ C_d = \text{const.} \]

Mass flow rate in a shallow recess
\( \frac{A_{orif}}{A_{inh}} = 2 \)

The restriction is ensured by the inherent surface

\[ C_d = \text{const.} \]
Comparison CFD vs. Bulk Flow (Ps=7bar, Ω=0rpm)

**Static pressure in the axial direction**

![Graph showing static pressure comparison]

**Mach number in the axial direction**

![Graph showing Mach number comparison]

**Static temperature in the axial direction**

![Graph showing static temperature comparison]
How good is the equation of the mass flow rate for unsteady working conditions?

$$\dot{m}_{th} = C_d \frac{AP_s}{\sqrt{T_s}} Q_{ideal}(P_s, P_r, \gamma)$$

CFD analysis of the unsteady flow

Time variation of the film thickness:

$$h(t) = h_0 + h_i \sin(2\pi ft)$$

$$h_i \approx 10\% h_0$$
Up to 1kHz excitation frequency the orifice mass flow rate equation can be used.

For very excitation high excitation frequencies the recess pressure and the film thickness have a phase difference larger than 90°.
EXPERIMENTAL ANALYSIS

-Rotordynamic coefficients
The rotor/air bearings test rig

Rotordynamic coefficients are identified by using impact hammer excitations.
Measured dynamic coefficients
Measured vs. calculated dynamic coefficients

Are the predicted rotordynamic coefficients good enough?

• Unbalance responses
• Stability analyses
Measured vs. calculated unbalance response

Dynamic coefficients obtained with the « equivalent » restrictor are injected in a 4DOF rigid rotor model.
STABILITY ANALYSIS

The whirl/whip instability
Linear stability of a 2DOF rotor

- Critical mass \( M_{cr} = \left( K + k \frac{c}{C} \right) \frac{C^2}{k^2} \)
- Whirl frequency \( \omega_{cr} = k/C \)

Critical mass estimated with measured dynamic coefficients
Whip at $P_{\text{supply}} = 2.5$ bar and 52 krpm

Rotation frequency [Hz]

Response frequency [Hz]

<0.5X

1X

(Very) localized wear of 5µm depth

Very rapid impact that caused damages!
STABILITY ANALYSIS (2)

The « pneumatic hammer » problem
A simplified problem: the circular aerostatic thrust bearing

Results of the lumped parameter, static analysis (quasi-analytic solution, Licht, 1967):

Static load: \[ W_0 = \pi R^2 \left\{ \frac{P_r + \sqrt{\pi} P_s}{2} C(e^{P_r/C} - e^{P_s/C}) \right\} \]

Dynamic analysis: first order solution (small perturbation)

\[ P_r(t) = P_{r0} + P_{r1}(t) \quad h(t) = h_0 + h_1(t) \]

\[ W(t) = W_0 + W_1(t) \quad \dot{M}(t) = \dot{M}_0 + \dot{M}_1(t) \]

Recesses mass flow rate balance eq.

\[
\frac{\partial M}{\partial t} = \dot{M}_0(t) - \dot{M}_s(t) \rightarrow P_{r1} \\

Z_b(s) = -\frac{W_1(s)}{h_1(s)} = K_0 \frac{1 + \tau_1 s}{1 + \tau_2 s} \quad Z_b(\omega) = K(\omega) + j\omega C(\omega) \\

K(\omega) = K_0 \frac{1 + \omega^2 \tau_1 \tau_2}{1 + \omega^2 \tau_2^2} \\
C(\omega) = K_0 \frac{\tau_1 - \tau_2}{1 + \omega^2 \tau_2^2} \]

Mass flow rate

\[ \dot{M}_b = \frac{h_0 (P_r^2 - P_s^2)}{12 \mu \tau_s \ln(R_b/R_r)} \quad \dot{M}_o = C_d A_o P_s \sqrt{\ldots} \]

\[ \dot{M}_b(h_0) = M_{0\text{ yields}} h_0 \]
Characteristics of the aerostatic **thrust** bearing

Negative damping and the accompanying « pneumatic hammer » instability are controlled by:
- Recess depth
- The ratio $P_{\text{recess}}/P_{\text{supply}}$

Fuller (1984, pg. 534) suggests operating at high $P_{\text{recess}}/P_{\text{supply}}$ by using large size orifice restrictors.
Theoretical investigation of the « pneumatic hammer » instability in the aerostatic bearing

- Centered bearing
- Zero rotation speed
- Different diameters of the restrictor orifice: 50%, 68%, 84% and 100% of Dorifice

Theoretical analysis:
- 1st step: steady CFD analysis
- 2nd step: « bulk flow » calculations of K and C (M→∞)
2nd step: «bulk flow» calculations of dynamic coefficients

- Centered bearing and zero rotation speed
- Mass \( \rightarrow \infty \) i.e. (almost) zero excitation frequency

Zone of pneumatic hammer instability
A new aerostatic bearing with dismounting orifices of different diameters was designed and manufactured.
Experimental dynamic coefficients

- Centered bearing
- Zero rotation speed
- Excitations applied by a single shaker in the range 250…450 Hz
- Results obtained with three diameters of the restrictor orifice: 68%, 84% and 100% of Dorifice
Results obtained for the smallest restrictor, 50\%Dorifice

« Pneumatic hammer » instabilities have a specific signature with a dominant frequency and its entire multiples
CONCLUSIONS

Is the work on hybrid aerostatic bearing finished?

- The accuracy of the predictions is satisfactory but can be still criticized; not all configurations are yet tested.

- Theoretical predictions are not very time consuming and enable parametric studies but very extensive theoretical analyses are still a problem (for example analyzing the impact of manufacturing tolerances)

- The experimental activity is still going on and is continuously consolidated.

Will the hybrid hydro/aerostatic bearing finally replace ball bearings?

Acknowledgements

SNECMA Space Propulsion Division, Centre National d’Etudes Spatiales

Laurent RUDLOFF, Amine HASSINI, Franck BALDUCCHI, Lassad AMAMI, Sylvain GAUDILLERE
Thank you for your attention